

0.1 Definition 1: Random Variables

Consider the following variables:

1. Distance from a bird's nest to the nearest one (meters).
2. Life time of certain machine (years).

Such quantities are called **continuous random variables**. Their values may be measured only to the nearest integer, but they actually range over an interval of real numbers.

0.2 Definition 2: Discrete Probability Function of a Random Variable

A function defined over a finite or an infinite discrete set of a random variable $\{x_1, x_2, \dots, x_n\}$ or $\{x_1, x_2, \dots\}$, respectively, is a probability function if

$$\text{i) } 0 \leq f(x_i) \leq 1 \quad i = 1, 2, \dots, \quad \text{and} \quad \text{ii) } f(x_1) + f(x_2) + \dots + f(x_n) = 1$$

0.3 Definition 3: Probability Density Function

A function f is a **probability density function** of a random variable x in the interval $[a, b]$ if

$$\text{i) } f(x) \geq 0 \quad \text{for all } x \text{ in } [a, b] \quad \text{ii) } \int_a^b f(x) dx = 1$$

0.4 Definition 4

Probability that a random variable x is in an interval $[c, d]$ contained in $[a, b]$

$$P(c \leq x \leq d) = \int_c^d f(x) dx$$

0.5 Proposition 1

If the domain of the probability density f is $(-\infty, \infty)$ then,

1. $P(x \leq b) = P(x < b) = \int_{-\infty}^b f(x) dx$
2. $P(x \geq a) = P(x > a) = \int_a^{\infty} f(x) dx$
3. $P(-\infty < x < \infty) = \int_{-\infty}^{\infty} f(x) dx$

0.6 Example 1

(Problem #36 in Section 11.1) Time required for a person to learn a task has a probability density function given by

$$f(x) = \frac{8}{7(x-2)^2}, \quad x \in [3, 10]$$

Find the probabilities that a person will learn the task in

- i) less than 4 minutes
- ii) more than 5 minutes.

0.7 Definition 5: Mean or Expected Value for Random Variable

For a discrete random variable

$$\mu = x_1p(x_1) + x_2p(x_2) + x_3p(x_3) + \dots x_np(x_n) = \sum_{i=1}^n x_ip(x_i).$$

For a continuous random variable

$$E(x) = \mu = \int_a^b xf(x) dx$$

0.8 Definition 6: Variance and Standard Deviation for a Random Variable

The Variance and Standard Deviation measure how spread are the values of a random variable

Discrete case:

$$Var(x) = \sum_{i=1}^n (x_i - \mu)^2 p(x_i), \quad \sigma = \sqrt{Var(x)}$$

Continuous case:

$$Var(x) = \int_a^b (x - \mu)^2 f(x) dx, \quad \sigma = \sqrt{Var(x)}$$

0.9 Proposition 2: Alternative Formula for the Variance

$$Var(x) = \int_a^b x^2 f(x) dx - \mu^2.$$